Army Research Laboratory

The Influence of Armor Material Parameters on the Penetration by Long-Rod Projectiles

by William P. Walters and Cyril L. Williams

ARL-RP-129 **September 2006**

**A reprint from the *2006 ASME Pressure Vessels and Piping Conference Proceedings*,
23–27 July 2006, Vancouver, BC.**

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED (From - To)
September 2006	Reprint	January 2006–July 2006

4. TITLE AND SUBTITLE
The Influence of Armor Material Parameters on the Penetration by Long-Rod Projectiles

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
William P. Walters and Cyril L. Williams

5d. PROJECT NUMBER
AH80

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
ATTN: AMSRD-ARL-WM-TC
Aberdeen Proving Ground, MD 21005-5066

8. PERFORMING ORGANIZATION REPORT NUMBER
ARL-RP-129

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
A reprint from the *2006 ASME Pressure Vessels and Piping Conference Proceedings*, 23–27 July 2006, Vancouver, BC.

14. ABSTRACT
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15. SUBJECT TERMS
penetration, Tate equations, perturbation theory

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			William Walters
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL	12	19b. TELEPHONE NUMBER (*Include area code*) 410-278-6062

THE INFLUENCE OF ARMOR MATERIAL PARAMETERS ON THE PENETRATION BY LONG-ROD PROJECTILES

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ABSTRACT

The Alekseevski-Tate equations have long been used to predict the penetration, penetration velocity, rod velocity, and rod erosion of long-rod projectiles or kinetic-energy penetrators [1]. These nonlinear equations were originally solved numerically, then by the exact analytical solution of Walters and Segletes [2, 3]. However, due to the nonlinear nature of the equations, the penetration was obtained implicitly as a function of time, so that an explicit functional dependence of the penetration on material properties was not obtained. Walters and Williams [4, 5, 6] obtained the velocities, length, and penetration as an explicit function of time by employing a perturbation solution of the non-dimensional Alekseevski-Tate equations. Algebraic equations were obtained for a third-order perturbation solution which showed excellent agreement with the exact solution of the Tate equations for tungsten heavy alloy rods penetrating a semi-infinite armor plate. The current paper employs this model to rapidly assess the effect of increasing the impact velocity of the penetrator and increasing the armor material properties (density and target resistance) on penetration. This study is applicable to the design of hardened targets.

INTRODUCTION

The Alekseevski-Tate Equations [1] hereafter referred to as the Tate equations, have long been employed to predict the penetration of long-rod, kinetic-energy penetrators impacting targets. Typically, these equations are solved by numerical integration techniques or by using the exact solution developed by Walters and Segletes [2, 3]. However, due to the nonlinear nature of these equations, the exact solution yields the penetration as an ***implicit*** function of time. An accurate, ***explicit*** solution to these equations, for each of the pertinent variables (penetration velocity, penetrator length, penetrator velocity, and penetration depth) was obtained by Walters and Williams [4, 5, 6] using a perturbation technique. The analytical nature of the resulting equations provides a simple (i.e., non-numerical) solution of the governing equations and clearly reveals the interplay of the various terms in the governing equations. This provides insight as to the importance of specific penetrator and target material properties. A perturbation solution was obtained for the following equation set [1, 2]:

$$(\rho_P/2)(v-u)^2 + Y_P = (\rho_T/2)u^2 + R_T \tag{1}$$

$$\frac{dv}{dt} = \frac{-Y_P}{l\rho_P} \tag{2}$$

$$\frac{dl}{dt} = u - v \tag{3}$$

$$u = \frac{dp}{dt} \text{ or } p = \int u\,dt \tag{4}$$

In these equations, v is the penetrator (or rod) velocity, u is the penetration velocity, p is the penetration, l is the penetrator length, t is the time after impact, R_T is the target strength term, Y_P is the penetrator strength term, ρ represents the density, where the subscript P stands for penetrator and subscript T represents the target. First, the equations are normalized and the method of normalization will depend on the input conditions; namely, the ρ values, the initial velocity, and the strength terms. For the usual case of interest to ballisticians studying kinetic- energy penetrators impacting armor targets, the following normalization parameters are introduced:

$$V = v/V_i \tag{5a}$$

$$U = u/V_i \tag{5b}$$

$$\lambda = l/L \tag{5c}$$

$$\tau = \beta t \tag{5d}$$

$$\mu^2 = \rho_T / \rho_P \tag{6a}$$

$$\beta = \left(\frac{\mu}{1+\mu}\right)\frac{V_i}{L} \tag{6b}$$

$$\alpha = \frac{R_T - Y_P}{Y_P} \tag{6c}$$

$$\varepsilon = \frac{Y_P}{\rho_P V_i^2} \tag{6d}$$

$$P = p/L \tag{6e}$$

where V_i is the impact velocity, L is the initial penetrator length, and V, U, P, λ, and τ are the dimensionless variables, while μ ,α, β, and ε are the dimensionless constants. The parameter β is used to normalize the time. The constant ε is the perturbation parameter.

The normalized equations become

$$(V - U)^2 - 2\alpha\varepsilon = \mu^2 U^2 \tag{7}$$

$$\frac{dV}{d\tau} = -\frac{\varepsilon}{\lambda}\left(\frac{1+\mu}{\mu}\right) \tag{8}$$

$$\frac{d\lambda}{d\tau} = \left(\frac{1+\mu}{\mu}\right)(U - V) \tag{9}$$

$$P = \left(\frac{1+\mu}{\mu}\right)\int_0^\tau U d\tau \tag{10}$$

The perturbation method [4, 5, 6] involves letting

$$V = V_0 + \varepsilon V_1 + \varepsilon^2 V_2 + \varepsilon^3 V_3 + \ldots\ldots\ldots\ldots \tag{11}$$

$$U = U_0 + \varepsilon U_1 + \varepsilon^2 U_2 + \varepsilon^3 U_3 + \ldots\ldots\ldots \tag{12}$$

$$\lambda = \lambda_0 + \varepsilon\lambda_1 + \varepsilon^2 \lambda_2 + \varepsilon^3 \lambda_3 + \ldots\ldots\ldots\ldots \tag{13}$$

$$P = P_0 + \varepsilon P_1 + \varepsilon^2 P_2 + \varepsilon^3 P_3 + \ldots\ldots\ldots\ldots. \tag{14}$$

and the perturbation parameter $\varepsilon = \frac{Y_P}{\rho_P V_i^2} << 1$ is chosen.

For example, if a third-order perturbation solution is obtained, terms of the order of ε to the fourth power and higher are neglected. Hence, the accuracy of the solution depends on the magnitude of ε and the number of terms in the above equation set.

One can substitute the expressions for V, U, λ, and P (Eqn. 11-14) into the nondimensional equation set and obtain, to order zero (considering only terms involving ε^0) , the following set:

$$(V_0 - U_0)^2 = \mu^2 U_0^2 \tag{15}$$

$$\frac{dV_0}{d\tau} = 0 \tag{16}$$

$$\frac{d\lambda_0}{d\tau} = -\left(\frac{1+\mu}{\mu}\right)(V_0 - U_0) \tag{17}$$

$$P_0 = \left(\frac{1+\mu}{\mu}\right)\int_0^\tau U_0 d\tau \tag{18}$$

where the initial conditions at $\tau = 0$ are $V_0(0) = 1$, $\lambda_0(0) = 1$.
The solution is straight forward:

$$V_0 = 1 \tag{19}$$

$$U_0 = \frac{1}{1+\mu} \tag{20}$$

$$\lambda_0 = 1 - \tau \tag{21}$$

$$P_0 = \frac{\tau}{\mu} \tag{22}$$

Note that this solution is that of the traditional hydrodynamic limit, where neither the target and penetrator strengths appear, and only penetrator and target densities determine the solution. Thus, the perturbation solution introduces strength effects, and the validity of these perturbations depend on the strengths being small relative to the normalization parameter (penetrator density times the square of the impact velocity).

We proceed in a similar manner and next consider terms of order ε, etc. Then one can calculate V_0, V_1, V_2, V_3, and the corresponding U, P, and λ expressions. These expressions are given in Walters and Williams [4, 5, 6] through the third order. The emphasis of the current paper is to exploit the influence of the target parameters on the rod penetration so the final expression for penetration is given below. Analogous equations for the rod velocity, penetration velocity, and rod length may be found in [4, 5, 6].

$$P = P_0 + \varepsilon P_1 + \varepsilon^2 P_2 + \varepsilon^3 P_3 \tag{23}$$

where:

$$P_1 = -\frac{\Theta}{\mu}\left[\tau(1+\alpha) + \Pi \ln \Pi\right]$$

$$P_2 = \frac{\Theta^2}{\mu}\left[\begin{array}{l}\left(3+\alpha+\dfrac{\alpha^2}{2}-\dfrac{2\alpha}{\mu}-\dfrac{\alpha^2}{2\mu}\right)\Pi - \Omega \ln \Pi - \\ \left(2+\alpha-\dfrac{\alpha}{\mu}\right)\Pi \ln \Pi + \dfrac{1}{2}\Pi(\ln \Pi)^2\end{array}\right] + C_{P2}$$

$$P_3 = -\frac{\Theta^3}{\mu}\left[\begin{array}{l}\left(\xi-\dfrac{\Omega^2}{2}\right)\Pi + \omega \Pi \ln \Pi + \dfrac{\Omega^2}{2\Pi} + \vartheta \ln \Pi + \\ \delta \Pi (\ln \Pi)^2 - \dfrac{\Omega}{2}(\ln \Pi)^2 + \dfrac{1}{2}\Pi(\ln \Pi)^3\end{array}\right] + C_{P3}$$

and

$$\Pi = 1-\tau$$

$$\Theta = \frac{(1+\mu)}{\mu}$$

$$\Omega = 1 - \frac{\alpha}{\mu}$$

$$\xi = -4 - 3\Omega^2 - 7\Omega + \frac{9\alpha^2}{2\mu} - 5\alpha - \frac{3\alpha^2}{2} - \frac{\alpha^3}{2\mu^2}(1-\mu)^2$$

$$\omega = \frac{7}{2} + 6\Omega + 3\alpha\Omega + \frac{3\Omega^2}{2} + \alpha + \frac{3\alpha^2}{2}$$

$$\vartheta = \Omega + \frac{5\Omega^2}{2} + \alpha - \frac{3\alpha^2}{2\mu} + \frac{1}{2}$$

$$\delta = -2\Omega - \frac{3}{2}(1+\alpha)$$

$$C_{P2} = -\frac{\Theta^2}{\mu}\left(3+\alpha+\frac{\alpha^2}{2}-\frac{2\alpha}{\mu}-\frac{\alpha^2}{2\mu}\right)$$

$$C_{P3} = \frac{\Theta^3 \xi}{\mu}.$$

RESULTS AND DISCUSSION

The normalization scheme used is deemed appropriate for cases of interest to ballisticians, namely heavy-metal, long-rod penetrators made of tungsten alloy or depleted-uranium penetrating a semi-infinite armor steel target. The third-order solution for P, with comparison to the exact Alekseevski-Tate equation solution is shown in Fig. 1. In this case, V_i = 2 km/s, L = 0.5 m, ρ_P = 17,600 kg/m^3, Y_P = 1.0 GPa, ρ_T = 7,800 kg/m^3, and R_T = 5.5 GPa. The perturbation parameter using these values is $\varepsilon = 0.0142$, which is much less than one. Terms with a coefficient of ε^4 and higher were neglected. The agreement for the third-order perturbation solution is excellent for penetration. Figure 1 is difficult to discern due to the fact that the curves for the first-order perturbation solution, the third-order perturbation solution, and Tate are in close agreement. As expected, the third-order perturbation solution is more accurate than the first order. In fact, at a time near the end of the penetration process, say 500 μs, the penetration depth deviates from the exact solution by 0.43% for the first-order perturbation solution and by 0.18% for the third-order perturbation solution.

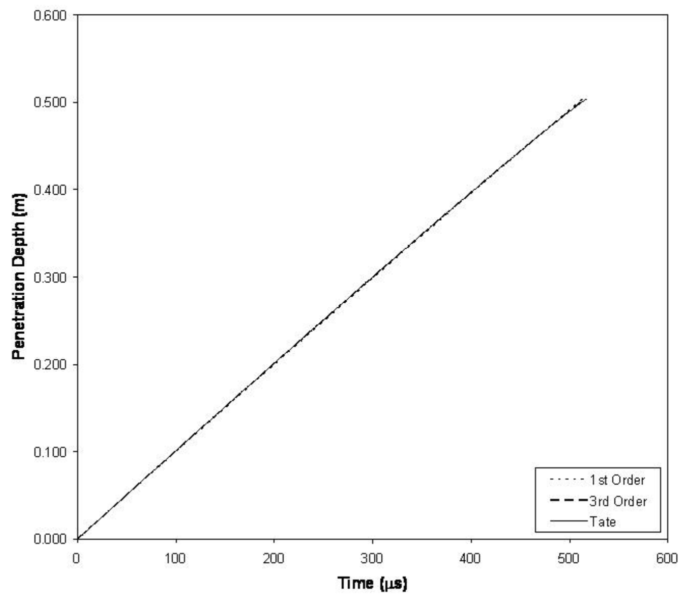

Fig.1. Penetration depth for a first-order perturbation solution, a third-order perturbation solution, and the exact Tate solution. This case is for a tungsten rod impacting a steel target at an impact velocity of 2000 m/s.

The perturbation equations can be used to determine the influence of material properties on penetration. The target properties are varied for a given penetrator. Thus, the perturbation parameter, which depends on the penetrator properties, remains small. Figure 2 plots the final penetration into semi-infinite RHA (rolled homogeneous armor) with the same input conditions as in Fig. 1 except the target resistance is allowed to vary from 1 to 20 GPa. Figure 3 returns the target resistance to its original value of 5.5 GPa and allows the density of the target to vary from 1,000 to 20,000 kg/m^3. If the target resistance is doubled for example, the penetration decreases by a 35.5%. If the target resistance is increased by

10%, the penetration decreases 4.4%, all else being equal. From Fig. 3, if the density is doubled from 7,800 to 15,600 kg/m^3, the penetration decreases by 21.0%. If the target density increases by 10%, the penetration decreases 3%. Other estimates as to the change in target resistance or density can be estimated from Fig. 2 and 3, respectively. Figure 4 shows the effect of increasing the impact velocity giving the characteristic S curve. The initial velocity required to achieve penetration is 715 m/s obtained from equation (1) when u = 0. The penetration increases by 24.2% if the impact velocity is increased from 2.0 km/s to 3.0 km/s. The pentration levels off at high velocities. Note that the third-order perturbation solution and the exact solution to the Tate equations are in good agreement. The first-order perturbation solution is not as accurate.

The purpose of this study is not to challenge the validity of the Tate equations. The Tate equations have been compared many times with experimental data and good agreement was obtained. Often the Y_P and R_T values are chosen to provide agreement with the experimental data. The intent is to indicate that an explicit solution is available that is accurate when compared to the exact or the numerical solution. Thus, a simple spreadsheet could be made to estimate the solutions very quickly for different material combinations.

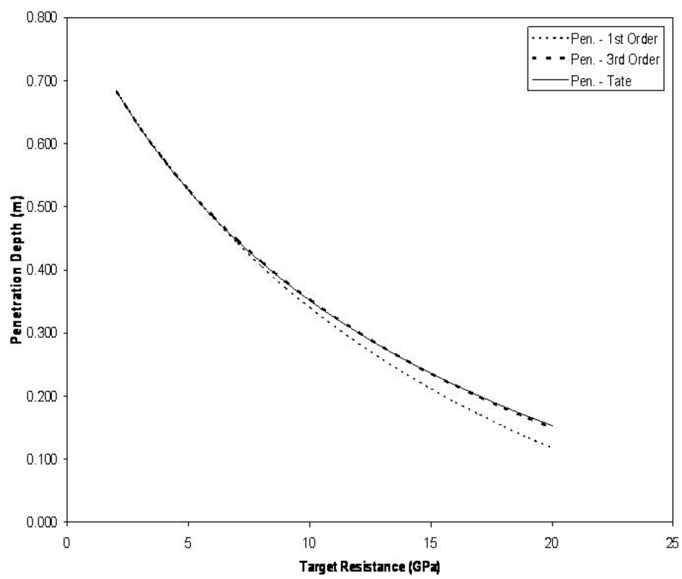

Fig.2. Final penetration into semi-infinite RHA target as a function of target resistance.

In general, a few comments are in order. All equations contain a term like ln (1-τ). Thus, a singularity occurs at $\tau = 1$ or $t = \frac{L(1+\mu)}{\mu V_i} = 625.5\mu s$ for the case described, and t must be less than this value. As this singularity is approached, deviation from the exact solution occurs. This occurs where the rod length and the velocity are both approaching zero simultaneously. Note that as the rod length approaches zero in the Alekseevski-Tate equations a singularity occurs. However, as long as the singularity is not approached, the solution is accurate.

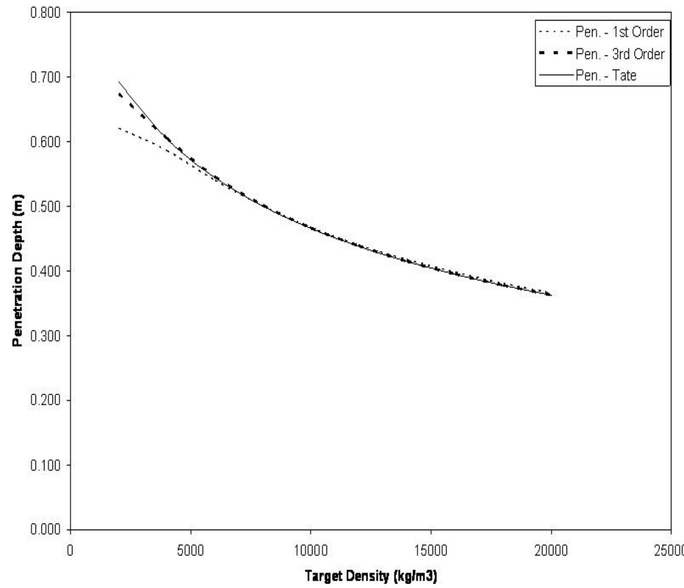

Fig.3. Final penetration into semi-infinite RHA target as a function of target density.

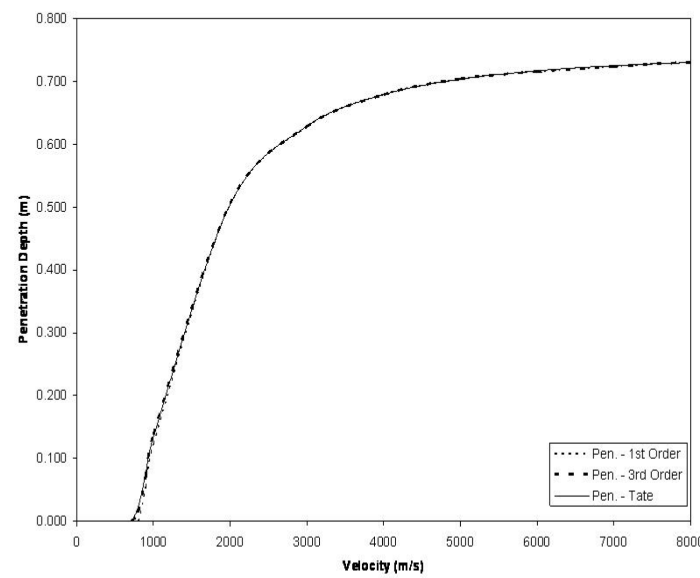

Fig.4. Final penetration into semi-infinite RHA target as a function of penetrator impact velocity.

As previously mentioned, the perturbation parameter must be much less than one. The value of ε used in the above equations implies a small penetrator strength or $Y_P << \rho_P V_i^2$. If this is not the case, for example, for a soft target (i.e., small R_T), the equations can be normalized with

$$V = v/V_i \tag{24a}$$

$$U = u/V_i \tag{24b}$$

$$\lambda = l/L \tag{24c}$$

$$P = p/L \quad (24d)$$

and the equation set becomes

$$(V-U)^2 + 2\alpha\varepsilon = \mu^2 U^2 \quad (25)$$

$$\frac{dV}{d\tau} = -\frac{\varepsilon}{\lambda}\left(\frac{1+\mu}{\mu}\right)\left(\frac{Y_p}{R_T}\right) \quad (26)$$

$$\frac{d\lambda}{d\tau} = \left(\frac{1+\mu}{\mu}\right)(U-V) \quad (27)$$

This is similar to our previous set of equations and thus follows the same solution scheme.

Note that the normalization scheme was obtained by dividing the first equation by ρ_p and defining μ^2 to be ρ_T/ρ_P. Alternately, one could divide the equation by ρ_T and define μ^2 to be ρ_P/ρ_T.
Other normalization schemes are possible, namely if the nondimensional variables are defined as

$$\lambda = l/L \quad (28a)$$

$$\mu = \rho_T/\rho_P \quad (28b)$$

$$K = \frac{Y_P}{\rho_P} \quad (28c)$$

$$\Sigma = \frac{2(R_T - Y_P)}{\rho_P} \quad (28d)$$

$$V = v/\sqrt{\Sigma} \quad (29a)$$

$$U = u/\sqrt{\Sigma} \quad (29b)$$

$$\tau = t\sqrt{\Sigma}/L \quad (29c)$$

$$P = p/L \quad (29d)$$

Note that $\Sigma > 0$ is required. Based on input, if this is not the case, one may reformulate Σ as $\frac{2(Y_p - R_T)}{\rho_T}$. Thus, for all cases the absolute value of Σ is used. The equation set becomes

$$(U-V)^2 = \mu U^2 + 1 \quad (30)$$

$$\frac{d\lambda}{d\tau} = U - V \quad (31)$$

$$\frac{\lambda dV}{d\tau} = \frac{-K}{\Sigma} = \frac{-Y_P}{2(R_T - Y_P)} \text{ or } \frac{dV}{d\tau} = \frac{\varepsilon}{\lambda} \quad (32)$$

where $\varepsilon = \frac{-K}{\Sigma} = \frac{-Y_P}{2(R_T - Y_P)}$ is chosen to be the perturbation parameter. Again, the above set of equations is very similar to the original set of equations and thus follows the same solution scheme. The normalization scheme or equation set chosen will depend on the input conditions (known initial values) and the requirement to keep $\varepsilon \ll 1$. Also, it is advantageous to make the time where the logarithmic singularity occurs ($\tau = 1$) as large as possible.

Conclusions

A third-order perturbation solution of the Alekseevski-Tate equations was used to assess the influence of certain target material properties on penetration into armor. Agreement with the exact solution is excellent for a tungsten rod impacting a semi-infinite steel target at a velocity of 2.0 km/s. In this study, the impact velocity was varied to yield the well-known S curve behavior. The target resistance was varied and shown to decrease penetration by 35.5% if the resistance was doubled. Also, the effect of increasing the target density was illustrated where doubling the target density decreased the penetration by 21%. In addition, alternate forms of the normalization of the pertinent equations are investigated to obtain a perturbation parameter much less than one for various penetration problems depending on the input conditions, namely the target and penetrator densities, strengths, and initial impact conditions. The current third-order solution is expressible as an algebraic equation, amenable to a spread sheet or simple calculator evaluation to accurately assess the effects of the various material properties appearing in the equations.

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